EXERCISE A

1. $2^3 = 8$

Base is 2, exponent is 3, result is 8.

$\log_2(8) = 3$

2. $10^{-3} = \frac{1}{1000}$

Base is 10, exponent is -3, result is $\frac{1}{1000}$

$\log_{10}(\frac{1}{1000}) = -3$

3. $2^{-3} = 0.125$

Base is 2, exponent is -3, result is 0.125.

$\log_2(0.125) = -3$

4. $64^{\frac{1}{3}} = 4$

Base is 64, exponent is $\frac{1}{3}$, result is 4.

$\log_{64}(4) = \frac{1}{3}$

5. $6^3 = 216$

Base is 6, exponent is 3, result is 216.

$\log_6(216) = 3$

6. $2^7 = 128$

Base is 2, exponent is 7, result is 128.

$\log_2(128) = 7$

7. $4^5 = 1024$

Base is 4, exponent is 5, result is 1024.

$\log_4(1024) = 5$

8. $9^3 = 729$

Base is 9, exponent is 3, result is 729.

$\log_9(729) = 3$

9. $x^3 = 50$

Base is x, exponent is 3, result is 50.

$\log_x(50) = 3$

10. $6^x = 12$

Base is 6, exponent is x, result is 12.

$\log_6(12) = x$

Exercise B

1. $e^{x} = 8$

1. Take the natural log of both sides: $x = \ln(8)$
2. Calculate the value: $\ln(8) \approx 2.07944$

Answer: Answer: $x = \ln(8) \approx 2.079$

2. $e^{3t} = 15$

1. Take the natural log of both sides: $3t = \ln(15)$
2. Divide by 3: $t = \frac{\ln(15)}{3} \approx \frac{2.70805}{3} \approx 0.90268$

Answer: Answer: $t = \frac{\ln(15)}{3} \approx 0.9027$

3. $6e^{x} = 350$

1. Divide by 6: $e^{x} = \frac{350}{6} = \frac{175}{3}$
2. Take the natural log: $x = \ln(\frac{175}{3}) \approx 4.06044$

Answer: Answer: $x = \ln(\frac{175}{3}) \approx 4.060$

4. $\ln x = 12$

1. Rewrite in exponential form: $x = e^{12}$
2. Calculate the value: $e^{12} \approx 162754.79$

Answer: Answer: $x = e^{12} \approx 162800$

5. $9 \ln x = 72$

1. Divide by 9: $\ln x = 8$
2. Exponentiate: $x = e^{8}$
3. Calculate: $x \approx 2980.95...$

Answer: Answer: Exact: e^{8}; 4 sig. figs: 2981

6. $e^{4x} = 20$

1. Take the natural log: $4x = \ln(20)$
2. Divide by 4: $x = \frac{\ln(20)}{4}$
3. Calculate: $x \approx 0.74893...$

Answer: Answer: Exact: $\frac{\ln(20)}{4}$; 4 sig. figs: 0.7489

7. $e^{5x} = 35$

1. Take the natural log: $5x = \ln(35)$
2. Divide by 5: $x = \frac{\ln(35)}{5}$
3. Calculate: $x \approx 0.71124...$

Answer: Answer: Exact: $\frac{\ln(35)}{5}$; 4 sig. figs: 0.7112

8. $5 \ln x = 75$

1. Divide by 5: $\ln x = 15$
2. Exponentiate: $x = e^{15}$
3. Calculate: $x \approx 3269017.37...$

Answer: Answer: Exact: e^{15}; 4 sig. figs: 3.269×10^{6}

Exercise B Part Two

1. $\log_a f = 2$ and $\log_a g = 6$

1. Convert to index notation: $f = a^2$ and $g = a^6$
2. Express a in terms of g: $a = g^{1/6}$
3. Substitute a into the equation for f: $f = (g^{1/6})^2 = g^{2/6} = g^{1/3}$

Answer: Answer: $f = g^{1/3}$

2. $\log_a 2f = 12$ and $\log_a g = 8$

1. Convert to index notation: $2f = a^{12}$ and $g = a^8$
2. Express a in terms of g: $a = g^{1/8}$
3. Substitute a into the equation for $2f$: $2f = (g^{1/8})^{12} = g^{12/8} = g^{3/2}$
4. Solve for f: $f = \frac{1}{2}g^{3/2}$

Answer: Answer: $f = \frac{1}{2}g^{3/2}$

3. $\log_a f = 5$ and $\log_{2a} g = 5$

1. Convert to index notation: $f = a^5$ and $g = (2a)^5 = 2^5a^5 = 32a^5$
2. From the first equation, $a^5 = f$
3. Substitute f into the second equation: $g = 32f$
4. Solve for f: $f = \frac{g}{32}$

Answer: Answer: $f = \frac{g}{32}$

4. $\log_a f = 12$ and $\log_{3a} 6g = 15$

1. Convert to index notation: $f = a^{12}$ and $6g = (3a)^{15} = 3^{15}a^{15}$
2. From the first equation, $a = f^{1/12}$
3. Substitute a into the second equation:
   $6g = 3^{15}(f^{1/12})^{15} = 3^{15}f^{15/12} = 3^{15}f^{5/4}$
4. Solve for f: $f^{5/4} = \frac{6g}{3^{15}}$
5. Raise both sides to the power of $4/5$: $f = \left(\frac{6g}{3^{15}}\right)^{4/5}$

Answer: Answer: $f = \left(\frac{6g}{3^{15}}\right)^{4/5}$

Exercise C

1. $3 \log_a 4$

1. Apply the power rule: $\log_a(4^3)$
2. Calculate $4^3 = 64$

Answer: Answer: $\log_a 64$

2. $\frac{1}{2} \log_g 16$

1. Apply the power rule: $\log_g(16^{1/2})$
2. Since $16^{1/2} = \sqrt{16} = 4$

Answer: Answer: $\log_g 4$

3. $\log 0.5$

1. Express 0.5 as a fraction: $\log(\frac{1}{2})$
2. Use the quotient rule: $\log(1) - \log(2)$
3. Since $\log(1) = 0$, the result is $-\log 2$

Answer: Answer: $-\log 2$

4. $\log \sqrt[3]{7}$

1. Express the radical as a fractional exponent: $\log(7^{1/3})$
2. Apply the power rule: $\frac{1}{3} \log 7$

Answer: Answer: $\frac{1}{3} \log 7$

5. $3 + \log_a 8$

1. Express 3 as a logarithm with base a: $3 = \log_a(a^3)$
2. Use the product rule: $\log_a(a^3) + \log_a 8 = \log_a(8a^3)$

Answer: Answer: $\log_a(8a^3)$

6. $\frac{2}{3} \ln 343 - 2 \ln 14$

1. Simplify the first term: $\ln(343^{2/3}) = \ln((\sqrt[3]{343})^2) = \ln(7^2) = \ln 49$
2. Simplify the second term: $\ln(14^2) = \ln 196$
3. Use the quotient rule: $\ln(\frac{49}{196}) = \ln(\frac{1}{4})$
4. This can be written as $\ln(2^{-2}) = -2 \ln 2$

Answer: Answer: $\ln(\frac{1}{4})$ or $-2 \ln 2$

7. $\log_a a^7$

1. Use the identity rule $\log_b(b^n) = n$

Answer: Answer: 7

8. $6 + \log_a \frac{1}{a^2} + \log_c \sqrt{c}$

1. Simplify $\log_a \frac{1}{a^2} = \log_a(a^{-2}) = -2$
2. Simplify $\log_c \sqrt{c} = \log_c(c^{1/2}) = \frac{1}{2}$
3. Combine the values: $6 - 2 + 0.5 = 4.5$

Answer: Answer: 4.5

Exercise D

1. $\log_7 11$

1. Write in terms of $\log_{10}$: $\dfrac{\log_{10} 11}{\log_{10} 7}$
2. Calculate: $\dfrac{1.041392685}{0.845098040} \approx 1.2323$

2. $\log_{11} 7$

1. Write in terms of $\log_{10}$: $\dfrac{\log_{10} 7}{\log_{10} 11}$
2. Calculate: $\dfrac{0.845098040}{1.041392685} \approx 0.81151$

3. $\log_3 5$

1. Write in terms of $\log_{10}$: $\dfrac{\log_{10} 5}{\log_{10} 3}$
2. Calculate: $\dfrac{0.698970004}{0.477121255} \approx 1.4650$

4. $\log_8 5$

1. Write in terms of $\log_{10}$: $\dfrac{\log_{10} 5}{\log_{10} 8}$
2. Calculate: $\dfrac{0.698970004}{0.903089987} \approx 0.77400$

5. $\log_6 25$

1. Write in terms of $\log_{10}$: $\dfrac{\log_{10} 25}{\log_{10} 6}$
2. Calculate: $\dfrac{1.397940009}{0.778151250} \approx 1.7965$

6. $\log_9 99$

1. Write in terms of $\log_{10}$: $\dfrac{\log_{10} 99}{\log_{10} 9}$
2. Calculate: $\dfrac{1.995635195}{0.954242509} \approx 2.0913$

Exercise E

1. $8^{3x} = 24$

1. Take the natural logarithm of both sides: $3x \cdot \ln(8) = \ln(24)$
2. Isolate x: $x = \dfrac{\ln(24)}{3 \cdot \ln(8)}$
3. Calculate: $x \approx \dfrac{3.17805}{3 \cdot 2.07944} \approx \dfrac{3.17805}{6.23832} \approx 0.5094$

   Rounding to 3 significant figures: **0.509**

2. $9^{x} = 35$

1. Take the natural logarithm of both sides: $x \cdot \ln(9) = \ln(35)$
2. Isolate x: $x = \dfrac{\ln(35)}{\ln(9)}$
3. Calculate: $x \approx \dfrac{3.55535}{2.19722} \approx 1.6181$

   Rounding to 3 significant figures: **1.62**

3. $7^{9x} = 56$

1. Take the natural logarithm of both sides: $9x \cdot \ln(7) = \ln(56)$
2. Isolate x: $x = \dfrac{\ln(56)}{9 \cdot \ln(7)}$
3. Calculate: $x \approx \dfrac{4.02535}{9 \cdot 1.94591} \approx \dfrac{4.02535}{17.51319} \approx 0.2298$

   Rounding to 3 significant figures: **0.230**

4. $16^{5x} = 82$

1. Take the natural logarithm of both sides: $5x \cdot \ln(16) = \ln(82)$
2. Isolate x: $x = \dfrac{\ln(82)}{5 \cdot \ln(16)}$
3. Calculate: $x \approx \dfrac{4.40672}{5 \cdot 2.77259} \approx \dfrac{4.40672}{13.86295} \approx 0.3178$

   Rounding to 3 significant figures: **0.318**

5. $25^{5x} = 360$

1. Take the natural logarithm of both sides: $5x \cdot \ln(25) = \ln(360)$
2. Isolate x: $x = \dfrac{\ln(360)}{5 \cdot \ln(25)}$
3. Calculate: $x \approx \dfrac{5.88610}{5 \cdot 3.21888} \approx \dfrac{5.88610}{16.0944} \approx 0.3657$

   Rounding to 3 significant figures: **0.366**

1. $\log_7 11$

1. Write in terms of $\log_{10}$: $\frac{\log_{10} 11}{\log_{10} 7}$
2. Calculate: $\frac{1.041392685}{0.845098040} \approx 1.23249$

Answer: Answer: $\frac{\log_{10} 11}{\log_{10} 7} \approx 1.23249$

2. $\log_{11} 7$

1. Write in terms of $\log_{10}$: $\frac{\log_{10} 7}{\log_{10} 11}$
2. Calculate: $\frac{0.845098040}{1.041392685} \approx 0.81141$

Answer: Answer: $\frac{\log_{10} 7}{\log_{10} 11} \approx 0.81141$

3. $\log_3 5$

1. Write in terms of $\log_{10}$: $\frac{\log_{10} 5}{\log_{10} 3}$
2. Calculate: $\frac{0.698970004}{0.477121255} \approx 1.46497$

4. $\log_8 5$

1. Write in terms of $\log_{10}$: $\frac{\log_{10} 5}{\log_{10} 8}$
2. Calculate: $\frac{0.698970004}{0.903089987} \approx 0.77398$

Answer: Answer: $\frac{\log_{10} 5}{\log_{10} 8} \approx 0.77398$

5. $\log_6 25$

1. Write in terms of $\log_{10}$: $\frac{\log_{10} 25}{\log_{10} 6}$
2. Calculate: $\frac{1.397940009}{0.778151250} \approx 1.79649$

Answer: Answer: $\frac{\log_{10} 25}{\log_{10} 6} \approx 1.79649$

6. $\log_9 99$

1. Write in terms of $\log_{10}$: $\frac{\log_{10} 99}{\log_{10} 9}$
2. Calculate: $\frac{1.995635195}{0.954242509} \approx 2.09133$

Answer: Answer: $\frac{\log_{10} 99}{\log_{10} 9} \approx 2.09133$

6. $5 \log_{10} x = 55$

1. $\log_{10} x = 55/5 = 11$
2. $x = 10^{11} = 1.000 \times 10^{11}$

7. $9 \log_{10} x = 36$

1. $\log_{10} x = 36/9 = 4$
2. $x = 10^{4} = 10000$

8. $8 \log_{10} x = 12$

1. $\log_{10} x = 12/8 = 1.5$
2. $x = 10^{1.5} \approx 31.62$

9. $3 \log_{10} x = 26$

1. $\log_{10} x = 26/3 \approx 8.6667$
2. $x = 10^{8.6667} \approx 4.642 \times 10^{8}$

10. $7 \log_{10} x = 19$

1. $\log_{10} x = 19/7 \approx 2.7143$
2. $x = 10^{2.7143} \approx 517.9$

11. $3 \log_{10} x = 55$

1. $\log_{10} x = 55/3 \approx 18.3333$
2. $x = 10^{18.3333} \approx 2.154 \times 10^{18}$

12. $11 \log_{10} x = 77$

1. $\log_{10} x = 77/11 = 7$
2. $x = 10^{7} = 1.000 \times 10^{7}$

13. $12 \log_{10} x = 45$

1. $\log_{10} x = 45/12 = 3.75$
2. $x = 10^{3.75} \approx 5623$

14. $17 \log_{10} x = 69$

1. $\log_{10} x = 69/17 \approx 4.0588$
2. $x = 10^{4.0588} \approx 11450$

15. $10 \log_{10} x = 85$

1. $\log_{10} x = 85/10 = 8.5$
2. $x = 10^{8.5} \approx 3.162 \times 10^{8}$

16. $5e^{3t} = 12$

1. Divide by 5: $e^{3t} = 2.4$
2. Take the natural log of both sides: $3t = \ln(2.4)$
3. Solve for t: $t = \dfrac{\ln(2.4)}{3} \approx 0.2918$

Answer: Answer: $t = \dfrac{\ln(2.4)}{3}$

17. $\ln(5x - 3) = 9$

1. Exponentiate both sides: $5x - 3 = e^9$
2. Add 3: $5x = e^9 + 3$
3. Divide by 5: $x = \dfrac{e^9 + 3}{5}$

Answer: Answer: $x = \dfrac{e^9 + 3}{5}$

18. $e^x + 15e^{-1} = 8$

1. Note that $e^{-1} = 1/e$. The equation is $e^x + \dfrac{15}{e} = 8$
2. Isolate e^x: $e^x = 8 - \dfrac{15}{e}$
3. Take the natural log: $x = \ln(8 - \dfrac{15}{e})$

Answer: Answer: $x = \ln(8 - \dfrac{15}{e})$

19. $6^{50-x} = 2^{50}$

1. Take $\ln$ of both sides: $(50 - x)\ln 6 = 50 \ln 2$
2. Divide by $\ln 6$: $50 - x = \dfrac{50 \ln 2}{\ln 6}$
3. Solve for x: $x = 50 - \dfrac{50 \ln 2}{\ln 6} = 50(1 - \dfrac{\ln 2}{\ln 6})$

Answer: Answer: $x = 50(1 - \log_6 2)$

20. $4^{x+1} = 3^{2x}$

1. Take $\ln$ of both sides: $(x + 1)\ln 4 = 2x \ln 3$
2. Expand: $x \ln 4 + \ln 4 = 2x \ln 3$
3. Rearrange: $\ln 4 = x(2 \ln 3 - \ln 4)$
4. Solve for x: $x = \dfrac{\ln 4}{2 \ln 3 - \ln 4} = \dfrac{\ln 4}{\ln 9 - \ln 4} = \dfrac{\ln 4}{\ln(9/4)}$

Answer: Answer: $x = \dfrac{\ln 4}{\ln 2.25}$

21. $\ln 5 + \ln x = 8$

1. Combine logs: $\ln(5x) = 8$
2. Exponentiate: $5x = e^8$
3. Solve for x: $x = \dfrac{e^8}{5}$

Answer: Answer: $x = \dfrac{e^8}{5}$

22. $4^{2x} - 17(4^x) = -16$

1. Let $u = 4^x$. The equation is $u^2 - 17u + 16 = 0$
2. Factor: $(u - 16)(u - 1) = 0$. So $u = 16$ or $u = 1$
3. Solve for x: $4^x = 16 \Rightarrow x = 2$; $4^x = 1 \Rightarrow x = 0$

Answer: Answer: $x = 0, 2$

23. $e^{4x} + 4e^{2x} + 5 = 0$

1. Let $u = e^{2x}$. The equation is $u^2 + 4u + 5 = 0$
2. Check the discriminant: $D = 4^2 - 4(1)(5) = 16 - 20 = -4$
3. Since the discriminant is negative, there are no real solutions.

Answer: Answer: No real solution.

24. Solve $9^{x-2} = 3^y$ and $\log_3 2x = 1 + \log_3 y$

1. Simplify first eq: $(3^2)^{x-2} = 3^y \Rightarrow 2x - 4 = y$

2. Simplify second eq:

   $$\log_3(2x) - \log_3(y) = 1 \Rightarrow \log_3(\frac{2x}{y}) = 1 \Rightarrow \frac{2x}{y} = 3^1 \Rightarrow 2x = 3y$$

3. Substitute $y = 2x - 4$ into $2x = 3y$:
   $2x = 3(2x - 4) \Rightarrow 2x = 6x - 12 \Rightarrow 4x = 12 \Rightarrow x = 3$

4. Solve for y: $y = 2(3) - 4 = 2$

Answer: Answer: $x = 3, y = 2$